E UNION AND INTERSECTION

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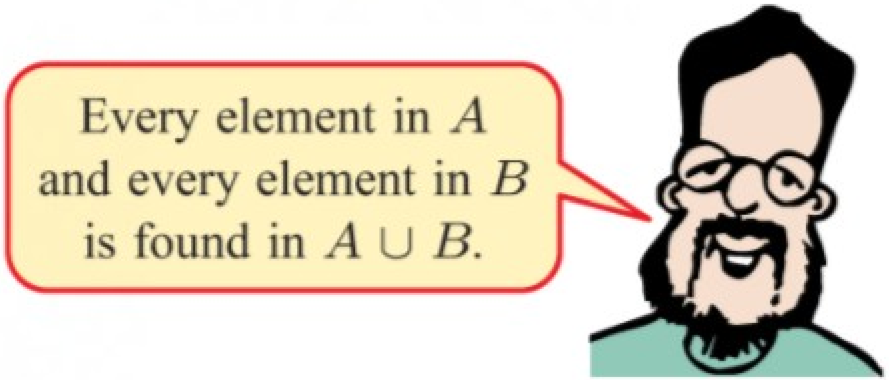
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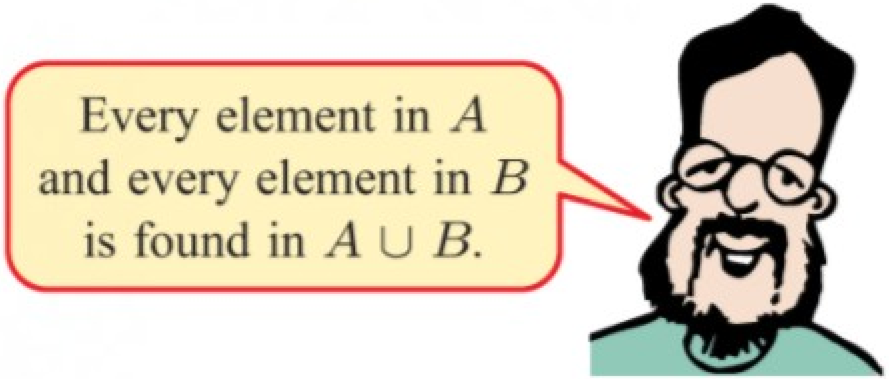
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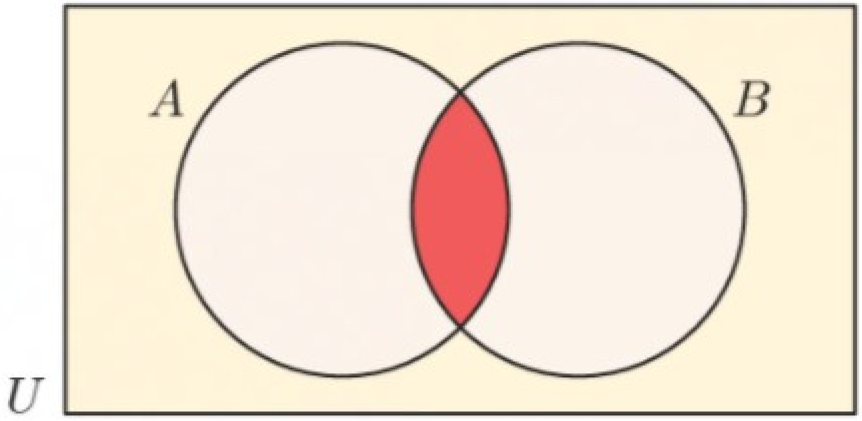
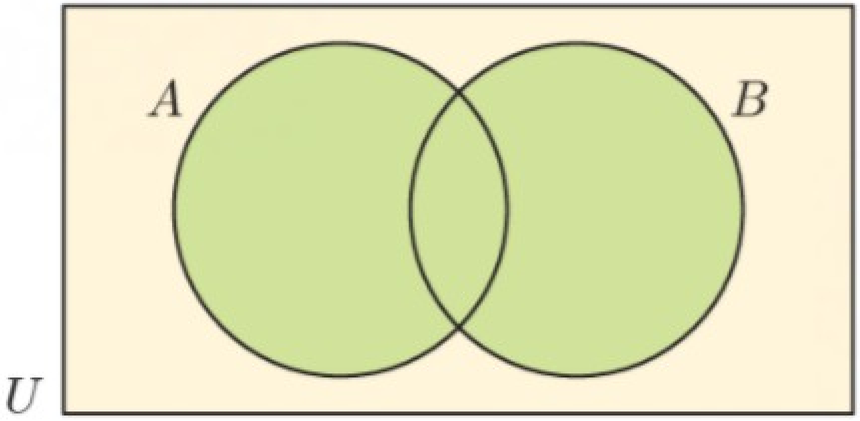


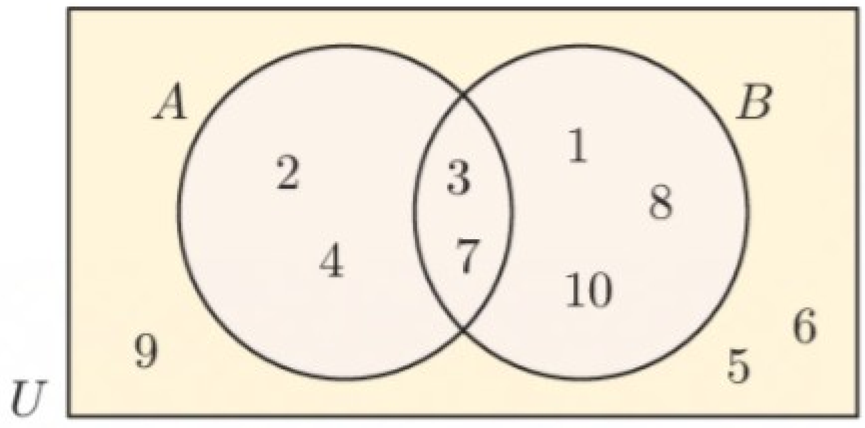
$A \cap B$ is shaded red.



$A \cup B$ is shaded green.

For example, the Venn diagram alongside shows

$A = \{2, 3, 4, 7\}$ and $B = \{1, 3, 7, 8, 10\}$.



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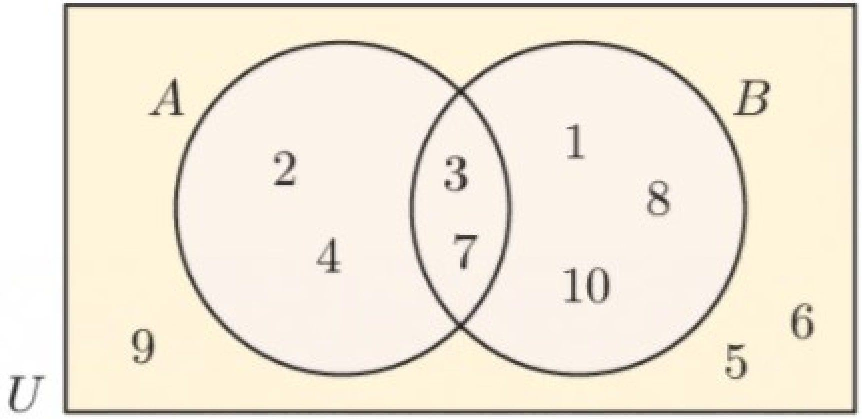


We can see that $A \cap B = \{3, 7\}$

and $A \cup B = \{1, 2, 3, 4, 7, 8, 10\}$.

Example 8

Self Tutor

Suppose $U = \{\text{positive integers} \leqslant 12\}$, $A = \{\text{primes} \leqslant 12\}$, and $B = \{\text{factors of } 12\}$.

a List the elements of the sets A and B.

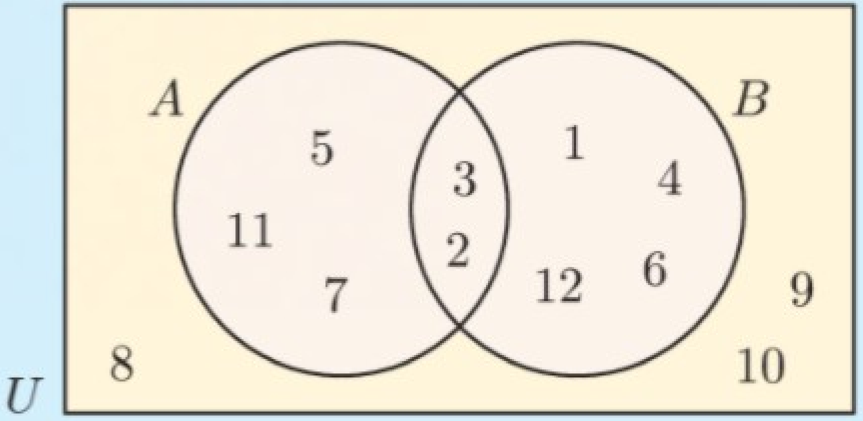
b Show the sets A, B, and U on a Venn diagram.

c List the elements in: **i** A' **ii** $A \cap B$ **iii** $A \cup B$

d Find: **i** $n(A \cap B)$ **ii** $n(A \cup B)$ **iii** $n(B')$

a $A = \{2, 3, 5, 7, 11\}$ and $B = \{1, 2, 3, 4, 6, 12\}$

b



c **i** $A' = \{1, 4, 6, 8, 9, 10, 12\}$ **ii** $A \cap B = \{2, 3\}$

iii $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 11, 12\}$

d **i** $n(A \cap B) = 2$ **ii** $n(A \cup B) = 9$

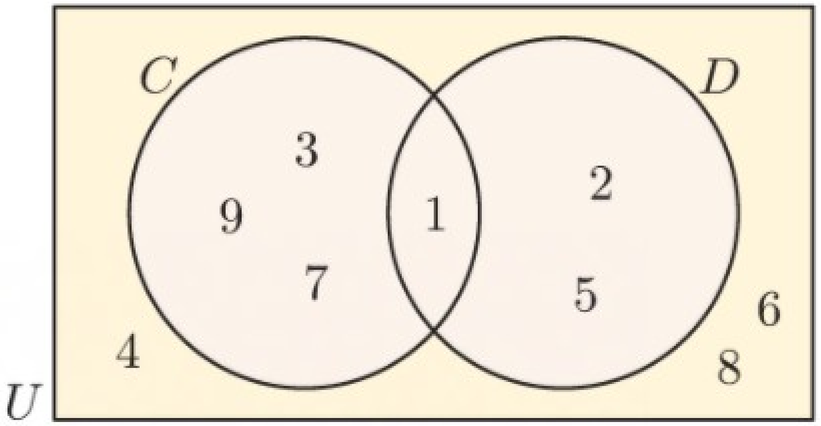
iii $B' = \{5, 7, 8, 9, 10, 11\}$, so $n(B') = 6$

Two sets are **disjoint** or **mutually exclusive** if they have no elements in common.

If A and B are disjoint then $A \cap B = \varnothing$.

EXERCISE 2E.1

1



a List the elements of set:

i C **ii** D **iii** U

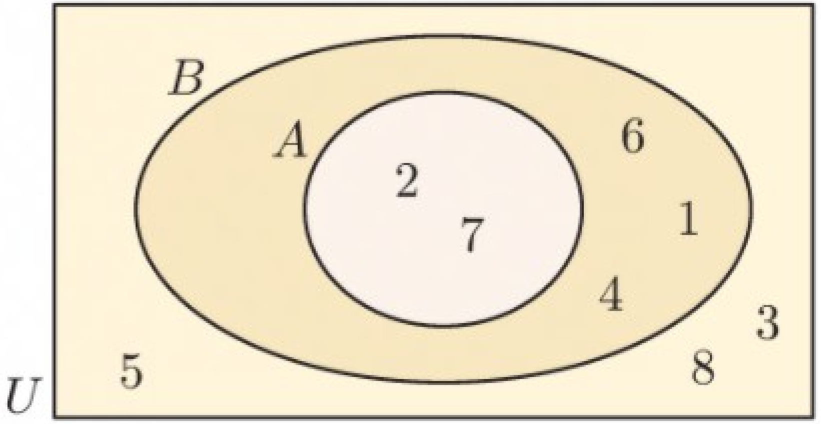
iv $C \cap D$ **v** $C \cup D$

b Find:

i $n(C)$ **ii** $n(D)$ **iii** $n(U)$

iv $n(C \cap D)$ **v** $n(C \cup D)$

2



a List the elements of set:

i A **ii** B **iii** U

iv $A \cap B$ **v** $A \cup B$

b Find:

i $n(A)$ **ii** $n(B)$ **iii** $n(U)$

iv $n(A \cap B)$ **v** $n(A \cup B)$

3 Consider $U = \{x \mid x \leqslant 12,\ x \in \mathbb{Z}^+\}$, $A = \{2, 7, 9, 10, 11\}$, and $B = \{1, 2, 9, 11, 12\}$.

- **a** Show these sets on a Venn diagram.
- **b** List the elements of: **i** $A \cap B$ **ii** $A \cup B$ **iii** B'
- **c** Find: **i** $n(A)$ **ii** $n(B')$ **iii** $n(A \cap B)$ **iv** $n(A \cup B)$

4 If A is the set of all factors of 36 and B is the set of all factors of 63, find:

- **a** $A \cap B$
- **b** $A \cup B$

5 If $X = \{\text{A, B, D, M, N, P, R, T, Z}\}$ and $Y = \{\text{B, C, M, T, W, Z}\}$, find:

- **a** $X \cap Y$
- **b** $X \cup Y$

6 Suppose $U = \{x \mid x \leqslant 30,\ x \in \mathbb{Z}^+\}$, $A = \{\text{factors of } 30\}$, and $B = \{\text{prime numbers} \leqslant 30\}$.

  a Find: **i** $n(A)$ **ii** $n(B)$ **iii** $n(A \cap B)$ **iv** $n(A \cup B)$

  b Show that $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

7 Simplify:

  a $X \cap Y$ for $X = \{1, 3, 5, 7\}$ and $Y = \{2, 4, 6, 8\}$

  b $A \cup A'$ for any set $A \in U$.

  c $A \cap A'$ for any set $A \in U$.

EXERCISE 2E.1

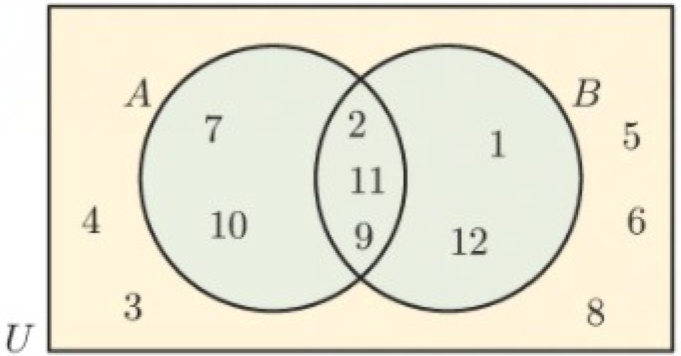
1 **a** **i** $C = \{1, 3, 7, 9\}$ **ii** $D = \{1, 2, 5\}$
iii $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ **iv** $C \cap D = \{1\}$
v $C \cup D = \{1, 2, 3, 5, 7, 9\}$

b **i** $n(C) = 4$ **ii** $n(D) = 3$ **iii** $n(U) = 9$
iv $n(C \cap D) = 1$ **v** $n(C \cup D) = 6$

2 **a** **i** $A = \{2, 7\}$ **ii** $B = \{1, 2, 4, 6, 7\}$
iii $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$ **iv** $A \cap B = \{2, 7\}$
v $A \cup B = \{1, 2, 4, 6, 7\}$

b **i** $n(A) = 2$ **ii** $n(B) = 5$ **iii** $n(U) = 8$
iv $n(A \cap B) = 2$ **v** $n(A \cup B) = 5$

3 **a**



b **i** $A \cap B = \{2, 9, 11\}$
ii $A \cup B = \{1, 2, 7, 9, 10, 11, 12\}$
iii $B' = \{3, 4, 5, 6, 7, 8, 10\}$

c **i** $n(A) = 5$ **ii** $n(B') = 7$
iii $n(A \cap B) = 3$ **iv** $n(A \cup B) = 7$

4 **a** $A \cap B = \{1, 3, 9\}$

b $A \cup B = \{1, 2, 3, 4, 6, 7, 9, 12, 18, 21, 36, 63\}$

5 **a** $X \cap Y = \{\text{B, M, T, Z}\}$

b $X \cup Y = \{\text{A, B, C, D, M, N, P, R, T, W, Z}\}$

6 **a** **i** $n(A) = 8$ **ii** $n(B) = 10$

iii $n(A \cap B) = 3$ **iv** $n(A \cup B) = 15$

b $$\begin{aligned} n(A) + n(B) - n(A \cap B) &= 8 + 10 - 3 \\ &= 15 = n(A \cup B) \end{aligned}$$

7 **a** $\varnothing$ **b** U **c** $\varnothing$